Multimedia Indexing and Retrieval

2. Classical machine Learning for multimedia indexing

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Learning

- Machine learning: learning from data.
- Unsupervised learning:
  - Without human intervention,
  - Simple data,
  - Automatic class extraction (clustering).
- Supervised learning:
  - With human intervention (annotation),
  - Labeled (or annotated) data
  - Classification (predefined classes),
  - Regression (continuous values).

Supervised learning

- A machine learning technique for creating a function from training data.
- The training data consist of pairs of input objects (typically vectors) and desired outputs.
- The output of the function can be a continuous value (regression) or a class label (classification) of the input object.
- The task of the supervised learner is to predict the value of the function for any valid input object after having seen a number of training examples (i.e. pairs of input and target output).
- To achieve this, the learner has to generalize from the presented data to unseen situations in a "reasonable" way.
- The parallel task in human and animal psychology is often referred to as concept learning (in the case of classification).
- Most commonly, supervised learning generates a global model that helps mapping input objects to desired outputs.

(http://en.wikipedia.org/wiki/Supervised_learning)

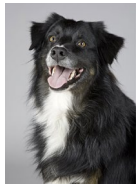
Learning a target function

- Target function: $f : X \rightarrow Y$

$$x \rightarrow y = f(x)$$

  - x : input object, e.g., color image
  - y : desired output, e.g., class label or image tag
  - X : set of valid input objects
  - Y : set of possible output values

f  = “cat”

f  = “dog”

f  = “car”

Set of possible color images:

$$X = \bigcup_{(w,h)\in\mathbb{N}^{*2}} [0,1]^{w\times h\times 3}$$

Set of possible image tags:

$$Y = \{\text{“cat”}, \text{“dog”} \ \ldots\}$$

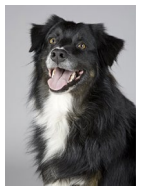
Learning a target function

- Target function: $f: X \rightarrow Y$

$$x \rightarrow y = f(x)$$

  - x : input object, e.g., color image
  - y : desired output, e.g., class label or image tag
  - X : set of valid input objects
  - Y : set of possible output values

$$f\left(\text{  }\right) = \begin{pmatrix} 0.90 \\ 0.04 \\ 0.01 \\ \dots \end{pmatrix} \begin{matrix} \leftarrow \text{“cat”} \\ \leftarrow \text{“dog”} \\ \leftarrow \text{“car”} \\ \leftarrow \dots \end{matrix}$$

$$f\left(\text{  }\right) = \begin{pmatrix} 0.07 \\ 0.88 \\ 0.02 \\ \dots \end{pmatrix}$$

$$f\left(\text{  }\right) = \begin{pmatrix} 0.02 \\ 0.03 \\ 0.86 \\ \dots \end{pmatrix}$$

Set of possible color images:

$$X = \bigcup_{(w,h)\in\mathbb{N}^{*2}} [0,1]^{w\times h\times 3}$$

Set of possible tag scores:

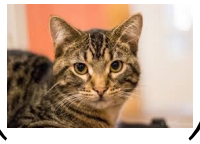
$$Y = \mathbb{R}^{|\{\text{“cat”},\text{“dog”}\dots\}|} = \mathbb{R}^{c}$$

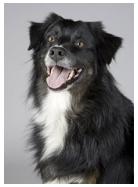
Learning a target function

- Target function: $f : X \rightarrow Y$

$$x \rightarrow y = f(x)$$

  - x : input object, e.g., image descriptor
  - y : desired output, e.g., class label or image tag
  - X : set of valid input objects
  - Y : set of possible output values

$$f\left(D\left(\text{  }\right)\right) = \begin{pmatrix} 0.90 \\ 0.04 \\ 0.01 \\ \dots \end{pmatrix}$$

$$f\left(D\left(\text{  }\right)\right) = \begin{pmatrix} 0.07 \\ 0.88 \\ 0.02 \\ \dots \end{pmatrix}$$

$$f\left(D\left(\text{  }\right)\right) = \begin{pmatrix} 0.02 \\ 0.03 \\ 0.86 \\ \dots \end{pmatrix}$$

Set of possible image descriptors:

$X = \mathbb{R}^d$ (or subset of it)

Set of possible tag scores:

$Y = \mathbb{R}^c$

D is a predefined and fixed function

from $\bigcup_{(w,h)\in\mathbb{N}^{*2}} [0,1]^{w\times h\times 3}$ to $\mathbb{R}^d$

Learning from training data

- Training data: $S = (x_i, y_i)_{(1 \le i \le I)}$
  - I : number of training samples

- Learning algorithm: $L : (X \times Y)^* \rightarrow Y^X$

$$S \rightarrow f = L(S)$$

$$\left((X \times Y)^* = \cup_{n \in N} (X \times Y)^n \right)$$

Y^X : set of all applications from X to Y

- Regression or classification system:

$$y = f(x) = [L(S)](x) = g(S, x)$$

Supervised learning

- Target function: $f : X \rightarrow Y$

$$x \rightarrow y = f(x)$$

  - x : input object (typically vector)
  - y : desired output (continuous value or class label)
  - X : set of valid input objects
  - Y : set of possible output values

- Training data: $S = (x_i, y_i)_{(1 \le i \le I)}$
  - I : number of training samples

- Learning algorithm: $L : (X \times Y)^* \rightarrow Y^X$

$$S \rightarrow f = L(S)$$

- Regression or classification system:

$$y = f(x) = [L(S)](x) = g(S, x)$$

Two types of functions

- Target function: $f : X \to Y$

$$x \to y = f(x)$$

  - maps input objects to desired outputs
  - often determined by a set of parameters
  - the function or its parameter are learnt from a training set

- Learning algorithm: $L : (X \times Y)^* \to Y^X$

$$S \to f = L(S)$$

  - maps training sets to target functions
  - often controlled by a set of hyper-parameters
  - hyper-parameters may be tuned on a validation set

Model based supervised learning

- Two functions, “train” and “predict”, cooperating via a Model

- General regression or classification system:

$$y = [L(S)](x) = g(S,x)$$

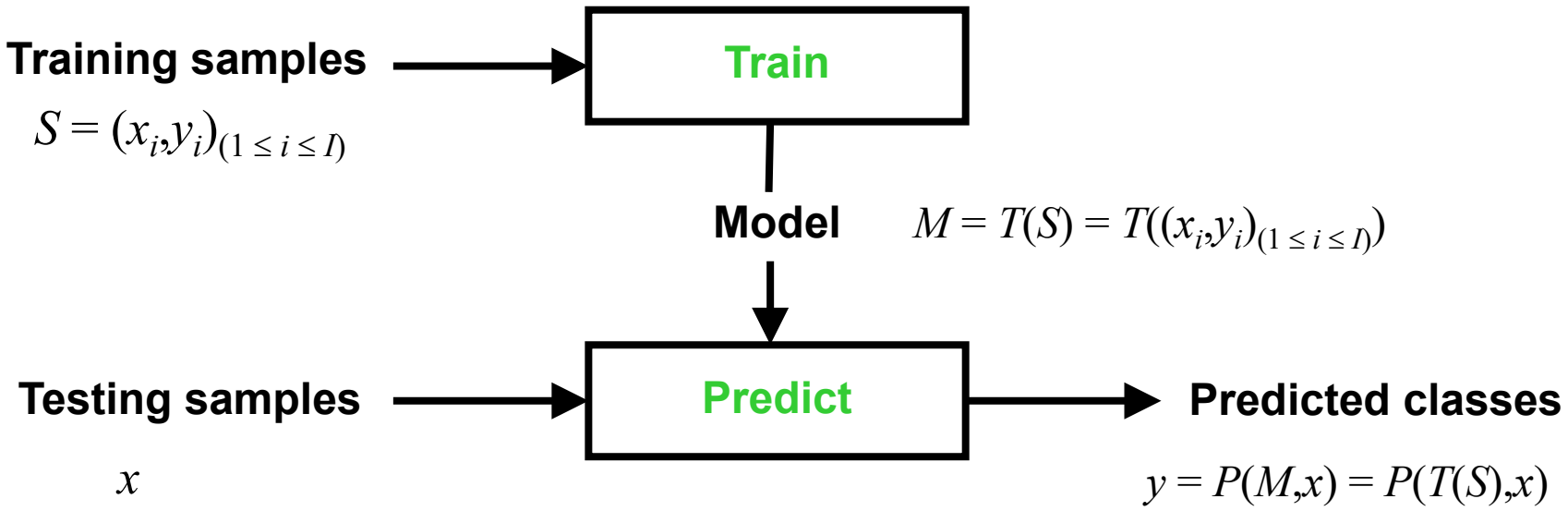
- Building of a model (train):

$$M = T(S)$$

- Prediction using a model (predict):

$$y = [L(S)](x) = g(S,x) = P(M,x) = P(T(S),x)$$

Supervised learning
Classification problem



Classification methods

- Gaussian Mixture Models
- Hidden Markov Models
- Decision trees
- Genetic algorithms
- Artificial neural networks
- K-nearest neighbor
- Linear discriminant analysis
- Support vector machines
- Minimum message length
- And many more.

k nearest neighbors (*k*-NN)

- No training : $M = T(S) = S$ (T = identity)
- Compute the distances from the unknown sample x to all the training samples x_i,
- Select the k closest x_i,
- Compute the class of *x* from the classes of the closest x_i's:
  - k = 1 : the class of x is the class of the closest x_i,
  - k is odd and there are only two classes : majority vote.
- k-NN is a non linear classifier and can easily model classes with very irregular shapes,

k nearest neighbors (*k*-NN)

- 1-NN is a simple and quite often excellent classifier, it is often chosen as a baseline for comparison between systems,
- 3-NN is more robust against isolated outliers,
- Improvement: weight class values according to the (inverse) distance to the query point
- May be slow for classification because of the need to compute the distances with all the training samples
- However a single NN search may be performed for many classifications at once (multi-label problem)
- May be used for indexing (off-line) or for search (on-line, “similarity search”)

Computation of distance for k-NN

- Euclidian distance, angle between vectors,
- Comparison between a query vector to all the vectors in the database (no pre-selection),
- “Small” number of dimensions (< 10) : clustering techniques, hierarchical search,
- “Medium” number of dimensions (~ 10+) : methods based on space partitioning,
- “Large” number of dimensions(>> 10) : no known method faster that a full linear scan,
- Reduction of the number of dimensions by Principal Component Analysis.
- Approximate Nearest Neighbors: LSH

Locality Sensitive Hashing (LSH)

- Hashing: store many data samples into a table of fixed length; data placed into “buckets”
- “Regular” Hashing: avoid collision for faster access, polynomial and multiples XOR functions; any type of data
- Locality preserving hashing: favor collisions of “close” samples into the same buckets; data from highly dimensional Euclidean space, multiple projection functions

LSH: Multiple projection functions

- Set of random directions
- Projection on the axes $\rightarrow$ one component per direction
- Split values on axes according to a data distribution (two, four, eight … intervals)
- One or more bits per direction (generally one)
- Concatenation for producing the bucket index
- Multiple projections: matrix vector multiplication

LSH: Use of multiple tables

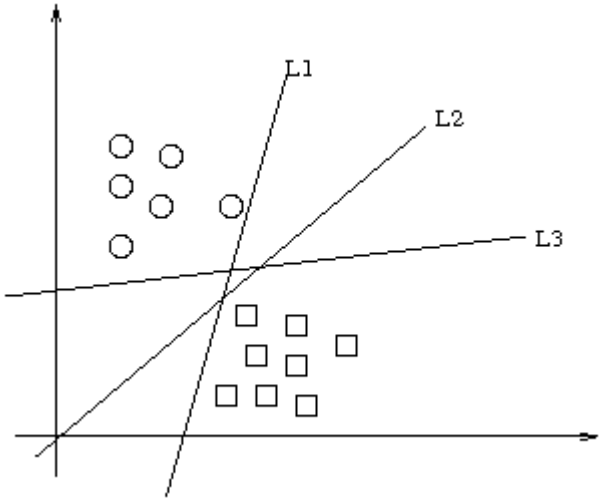
- Build many LSH tables
- For each table, select all the test samples that fall in the same bucket than the query sample
- Compute the Euclidean distance only for those samples
- Sort the test samples according to the Euclidean distances
- Euclidean distances are not approximate but some samples close to the query may not fall in the selected buckets
- The size and number of tables must be chosen so that enough and not too many samples are found for a query

LSH: Use of hamming distance

- Build binary codes (bucket index) as for one LSH table
- Hamming distance: number of bit locations in which the binary values differ: bitwise XOR followed by a count on 1 bits; modern processors have this as a single instruction
- Compute the Hamming distance between the query and all test samples: much faster than Euclidean distance
- Select samples with closest Hamming distance
- Compute the Euclidean distance only for those samples
- Similar to multiple tables from there

Support Vector Machines (SVM)

- Empirical risk minimization
- Linear classifier with maximum margin



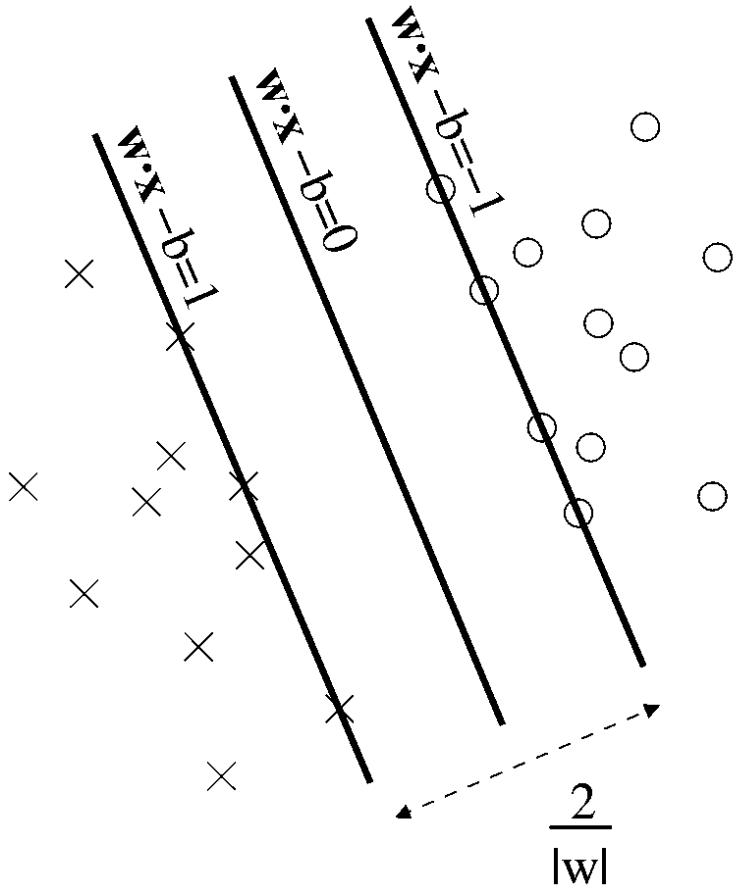
- The “kernel trick” permits non linear classification also with maximum margin and minimum empirical risk

SVM linear classification

- Maximum-margin hyperplanes for a SVM trained with samples from two classes.
- Samples along the hyperplanes are called the support vectors.
- The separated hyperplane is defined by:

$$w^T.x - b = 0$$

- The margin is $2/|w|$



SVM linear classification

- If the data is linearly separable:

  if $y_i = -1 : \; w^T.x_i - b \leq -1$ if $y_i = +1 : \; w^T.x_i - b \geq 1$

- This can be rewritten as:

  $$y_i.(w^T.x_i - b) \geq 1$$

- SVM problem primal form:

  Minimize: $\frac{1}{2}\|w\|^2$ subject to: $y_i.\left(w^T.x_i - b\right) \geq 1, \quad 1 \leq i \leq n.$

- SVM problem dual form: $w = \sum_{i=1}^{n} \alpha_i y_i x_i$

  maximize: $\sum_{i=1}^{n} \alpha_i - \sum_{i=1}^{n}\sum_{j=1}^{n} \alpha_i \alpha_j y_i y_j x_i^T x_j$ subject to $\alpha_i \geq 0$

  α_i 's are non zero only for the support vectors.

SVM linear classification

- Soft margin, primal form:

$$y_i.\left(w^T.x_i - b\right) \geq 1 \quad \rightarrow \quad y_i.\left(w^T.x_i - b\right) \geq 1 - \xi_i$$

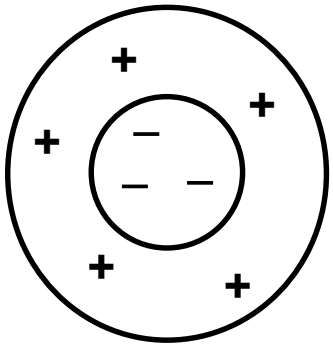
$$\min \frac{1}{2}\|w\|^2 \quad \rightarrow \quad \min\left(\frac{1}{2}\|w\|^2 + C\sum_{i=1}^{n}\xi_i\right)$$

- Dual form:

$$\alpha_i \geq 0 \quad \rightarrow \quad 0 \leq \alpha_i \leq C$$

- Allows for “misclassified” samples.

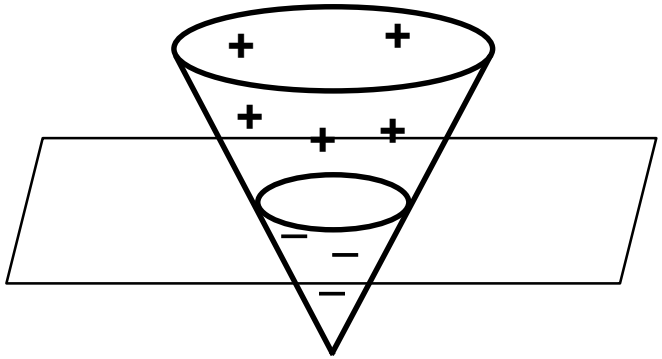
SVM non-linear classification



No linear separation in original space

- Kernel trick: projection on a cone (2D → 3D):

$$(x, y) \rightarrow \Phi(x, y) = \left(x, y, \sqrt{x^2 + y^2}\right)$$



Linear separation in im(Φ) space

SVM non-linear classification

- Decision function:

$$f(x) = \langle w | x \rangle - b = \left\langle \sum_{i=1}^{n} \alpha_i y_i x_i \middle| x \right\rangle - b = \left(\sum_{i=1}^{n} \alpha_i y_i \langle x_i | x \rangle \right) - b$$

- Quadratic form maximization:

$$\sum_{i=1}^{n} \alpha_i - \sum_{i=1}^{n} \sum_{j=1}^{n} \alpha_i \alpha_j y_i y_j \langle x_i | x_j \rangle$$

- Kernel trick: $\langle x_i | x_j \rangle \rightarrow \langle \Phi(x_i) | \Phi(x_j) \rangle = K(x_i, x_j)$

- Φ : possibly non-linear function, does not need to be computed, implicitly defined via the kernel (K) definition, linear separation in the im(Φ) space, may be non linear in the original space.

SVM non-linear classification

- Mercer condition : $K(x_i,x_j)$ must be definite positive.
- Common kernels:
  - Polynomial (homogeneous): $K(x,y)=(x.y)^d$
  - Polynomial (inhomogeneous): $K(x,y)=(x.y+1)^d$
  - Radial Basis Function: $K(x,y)=\exp\left(-\frac{\|x-y\|^2}{2\sigma^2}\right)$
  - Sigmoid: $K(x,y)=\tanh(\kappa x.y+c)$, for some (not every) $\kappa > 0$ and $c < 0$

SVM summary

- Maximization of the margin for linearly separable data
- Use of a dual form for finding support vectors and coefficients (convex optimization)
- Use of soft margin for “almost” linearly separable data
- Use of the “kernel trick” for non-linearly separable data
- Most commonly used kernel: $K(x,y) = e^{-\gamma\|x-y\|^2}$ $\rightarrow$ $f(x) = \sum_{i=1}^{i=I} \propto_i y_i e^{-\gamma\|x-x_i\|^2} + b$: weighted sum of Gaussians centered on the support samples (vectors)

Hyper-parameter tuning

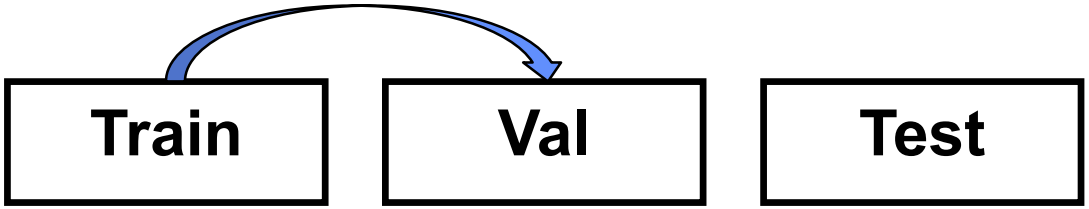
- Parameters:
  - Parameters of the model learnt from training data
  - e.g. values of the support vectors (x_i) and Lagrange coefficients (α_i) in SVMs
- "Hyper"-parameters:
  - Parameters that controls how the model (and "standard" parameters) are learnt
  - e.g. soft margin coefficient (C) in SVMs and the scale parameter in the RBF version (γ)
  - Possibly also class weights
  - Controls "how well" the classification algorithm generalizes from training data, especially the under fit versus over fit compromise

Hyper-parameter tuning, validation set

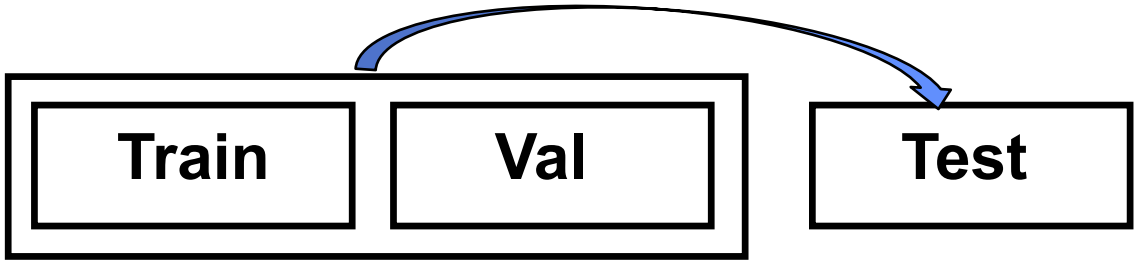
- A dataset used for training cannot be used for evaluation (over-fitting)
- Standard method: use different datasets for training and performance evaluation, each with annotated samples.
- Tuning of hyper-parameters on the test set is bad (over-fitting again)
- Good solution: use three datasets: train, val and test, all with annotated samples
- Train and evaluate several hyper-parameter values between train and val and then apply to test.

Hyper-parameter tuning, validation set

- Parameter tuning: selection of the optimal hyper-parameter combination by training on train and evaluating on val.

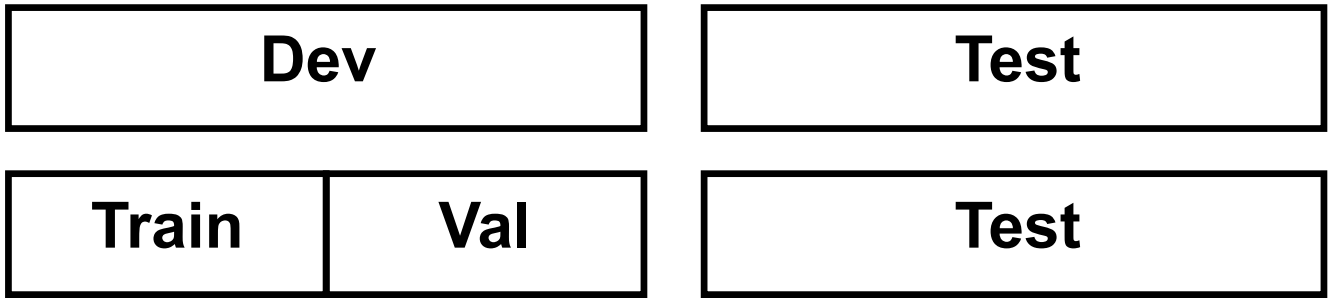


- Actual evaluation: keep the optimal hyper-parameter values, train on train+val and evaluate on test.

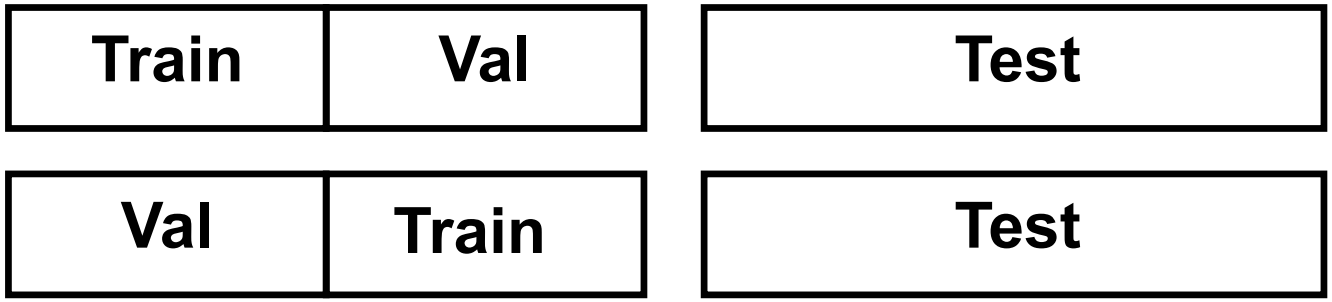


No validation set: split the training set

- Split into two equal parts, use first part as train and second part for validation ("one-fold" cross-validation)

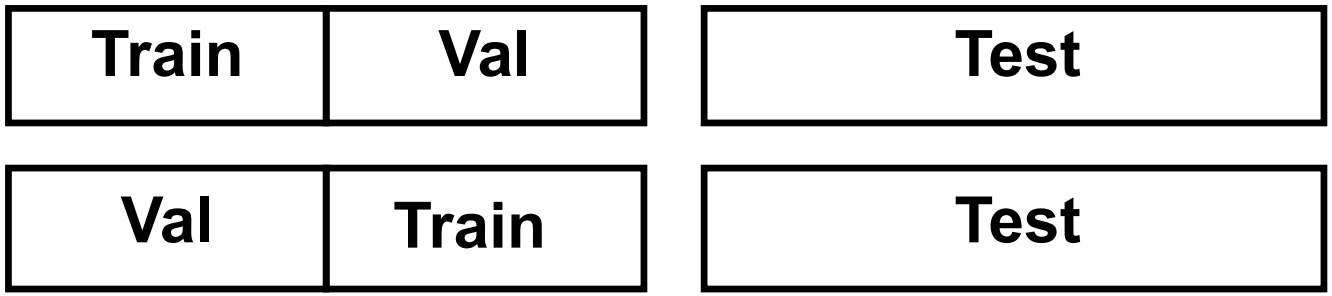


- Two-fold cross-validation



Two-fold cross-validation

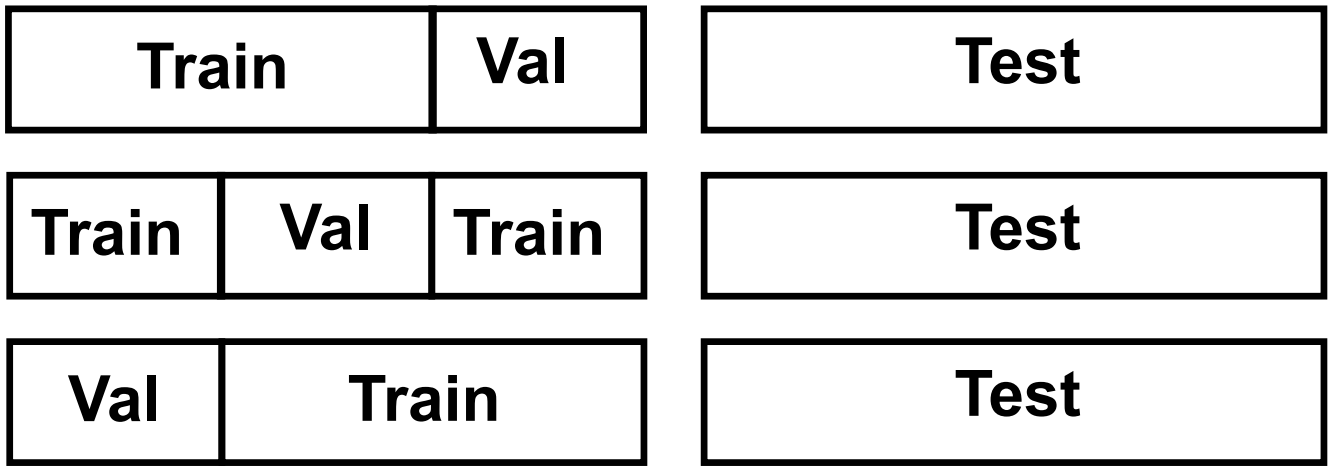
- Use two parts alternatively for training and validation



- The whole development set is used both for training and for evaluation during hyper-parameter tuning
- Tuning is done on MAP (hyper-parameters)
  - Either average the MAP on the two validations
  - Or compute a global MAP on the concatenated scores
- Training is done on half of the development set each time

N-fold cross-validation

- Use N parts of 1/N od the development set alternatively for validation and the complement ((N-1)/N) for training



- The whole development set is used both for training and for evaluation during hyper-parameter tuning
- Training is done on (N-1)/N of the development set each time, the greater N, the better.

Probabilized output

- SVM scores possibly ranges from $-\infty$ to $+\infty$
- Probabilities are expected to range from 0 to 1
- Sigmoid transform: $p(score) = 1/(1+e^{(A*score+B)})$
- Additional hint: among the samples within a small interval around p, a fraction of about p would have positive labels
- Platt's (1999) method: learn A and B by cross-validation to optimally satisfy the above hint
- Probability normalized outputs better for late fusion

- Linear SVM + sigmoid normalization ~ logistic regression